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COMPUTER PROGRAMS FOR H-FIELD, E-FIELD, AND COMBINED FIELD SOLUTIONS FOR BODIES OF REVOLUTION

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A computer program is given to implement the H-field, E-field, and Combined-field solutions given in Interim Technical Report RADC-TR-77-109 for a perfectly conducting body of revolution excited by an oblique plane wave incident field. The program consists of several subroutines and a main program. The main program calculates the electric current on the body of revolution and the bistatic scattering cross section per square wavelength. Some examples of computations are given and discussed.

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I. INTRODUCTION

Computer programs for E-field, H-field and combined field solutions for a perfectly conducting body of revolution excited by an obliquely incident plane wave electromagnetic field are described and listed in this report. The H-field solution is obtained by applying the method of moments to the H-field integral equation. The E-field solution is obtained by applying the method of moments to the E-field integral equation. The matrix equation for the combined field solution is a linear combination of the matrix equations for the H-field and E-field solutions. The general theory and the method of computation are given in reference [1]. Equations numbers drawn from reference [1] are preceded by 1-. For instance, (1-3) denotes equation three of reference [1].

The computer program subroutine YMAT calculates the square moment matrix for the H-field solution and is described and listed in Section II. The subroutine ZMAT calculates the moment matrix for the E-field solution and is described and listed in Section III. The subroutine YZ calculates the moment matrices for both the H-field and E-field solutions simultaneously and is listed in Section IV. The subroutine PLANE calculates the plane wave measurement vectors and is described and listed in Section V. As shown in [1], the plane wave excitation column vectors for the H-field and E-field solutions and hence also for the combined field solution can be expressed in terms of certain plane wave measurement vectors. A main program which uses YMAT, ZMAT, YZ, and PLANE to obtain the H-field solution, the E-field solution, and the combined field solution is described and listed in Section VII. The final section shows some examples of computations made with the program and gives a discussion of them.

---

[1] J. R. Mautz and R. F. Harrington, "H-Field, E-Field and Combined Field Solutions for Bodies of Revolution," Interim Technical Report RADC-TR-109, Rome Air Development Center, Griffiss Air Force Base, New York, March 1977

II. THE SUBROUTINE YMAT

A. Description:

The subroutine YMAT(NN,NP,NPHI,RH,ZH,X,A,Y) stores the matrix

$$[Y] = \begin{bmatrix} Y_n^{tt} & Y_n^{t\phi} \\ Y_n^{\phi t} & Y_n^{\phi\phi} \end{bmatrix} \quad (1)$$

appearing in the H-field matrix equation (1-17) by columns in Y. The elements of [Y] are given by (1-31). The input variables are defined in terms of variables appearing in reference [1] by

$$NN = n$$

$$NP = P$$

$$NPHI = N_\phi$$

$$RH(i) = k\bar{\rho}_i$$

$$ZH(i) = k\bar{z}_i$$

$$X(k) = x_k$$

$$A(k) = A_k$$

In summary, n denotes $e^{jn\phi}$ dependence, $(\bar{\rho}_i, \bar{z}_i)$, $i = 1,2,\ldots P$ are coordinates on the generating curve, the k which multiplies $\bar{\rho}_i$ and $\bar{z}_i$ is the propagation constant, and x_k and A_k are the abscissas and weights for the N_ϕ point Gaussian quadrature integration in ϕ.

Minimum allocations are given by

```
COMPLEX Y(4*N*N)
DIMENSION RH(NP), ZH(NP), X(NPHI), A(NPHI),
   D(NG), PD(NG), CR(NPHI), C1(NPHI),
   C2(NPHI), C3(NPHI)
COMMON RS(NG), ZS(NG), SV(NG), CV(NG), T(4*N)
```

where

$$NG = NP - 1$$

$$N = (NG-2)/2$$

The variables in common make the results of some intermediate calculations done in YMAT available to the subroutine PLANE described in Section V.

DO loop 57 puts kd_i of (1-28) in D, $k\rho_i$ of (1-27) in RS, kz_i of (1-27) in ZS, $\sin v_i$ in SV, $\cos v_i$ in CV, and $\frac{\pi}{k^2 d_i \rho_i}$ in PD. DO loop 68 puts (1-29) in T. With regard to (1-11) and (1-24) - (1-26), DO loop 25 puts $4 \sin^2(\frac{\phi_k}{2})$, $\pi A_k \sin^2(\frac{\phi_k}{2}) \cos(n\phi_k)$, $\frac{\pi}{2} A_k \cos \phi_k \cos(n\phi_k)$, and $\frac{\pi}{2} A_k \sin \phi_k \sin(n\phi_k)$ in CR, C1, C2, and C3 respectively where, as prescribed by (1-37),

$$\phi_k = \frac{\pi}{2} (x_k + 1)$$

DO loop 62 initializes Y.

The computation of (1-31) is sequential in s rather than ij. A computation sequential in s is preferable because it does not require storing $(Y1)_s$, $(Y2)_s$, $(Y3)_s$, or $(Y4)_s$ versus s. Unfortunately, (1-31) as it stands tacitly implies computation sequential in ij. An alternative form of (1-31) somewhat more suggestive of computation sequential in s is given by

$$\begin{bmatrix} (Y_n^{tt})_{pq} \\ (Y_n^{\phi t})_{pq} \\ (Y_n^{t\phi})_{pq} \\ (Y_n^{\phi\phi})_{pq} \end{bmatrix} = \sum_{i=2p-1}^{2p+2} T_{2p-2+i} \sum_{j=2q-1}^{2q+2} T_{2q-2+j} \begin{bmatrix} (Y1)_{ij} \\ (Y2)_{ij} \\ (Y3)_{ij} \\ (Y4)_{ij} \end{bmatrix} \qquad (2)$$

If (1-38) is true, then

$$\begin{aligned} ((Y_n^{tt})_{pq})_{ij} &= -((Y_n^{\phi\phi})_{qp})_{ji} \\ ((Y_n^{\phi t})_{pq})_{ij} &= -((Y_n^{\phi t})_{qp})_{ji} \\ ((Y_n^{t\phi})_{pq})_{ij} &= -((Y_n^{t\phi})_{qp})_{ji} \end{aligned} \qquad (3)$$

where the additional subscripts ij denote the contribution due to the ij^{th} term on the right-hand side of (2). From (3)

$$
\begin{aligned}
(Y_n^{tt})_{pq} &= ((Y_n^{tt})_{pq})_{i\leq j} - ((Y_n^{\phi\phi})_{qp})_{i\leq j} \\
(Y_n^{\phi t})_{pq} &= ((Y_n^{\phi t})_{pq})_{i\leq j} - ((Y_n^{\phi t})_{qp})_{i\leq j} \\
(Y_n^{t\phi})_{pq} &= ((Y_n^{t\phi})_{pq})_{i\leq j} - ((Y_n^{t\phi})_{qp})_{i\leq j} \\
(Y_n^{\phi\phi})_{pq} &= ((Y_n^{\phi\phi})_{pq})_{i\leq j} - ((Y_n^{tt})_{qp})_{i\leq j}
\end{aligned}
\tag{4}
$$

where the subscript notation $i\leq j$ denotes the sum of the contributions due to all terms for which $i < j$ plus half of the $i = j$ term on the right-hand side of (2). According to (4), it is sufficient to calculate only the $i\leq j$ terms on the right-hand side of (2). Inspection of the limits of summation in (2) shows that the first term on the right-hand side of (4) is zero if $p > q+1$ and that the second term is zero if $p < q-1$.

The ij^{th} term of the first of equations (2) contributes

$$
T_{2(p+k-1)+i} \quad T_{2(q+\ell-1)+j} (Y1)_{ij} \quad \text{to} \quad (Y_n^{tt})_{(p+k)(q+\ell)} \,, \quad \begin{matrix} k=1,2 \\ \ell=1,2 \end{matrix}
\tag{5}
$$

where
$$
\begin{aligned}
p &= [(i+1)/2]-2 \\
q &= [(j+1)/2]-2
\end{aligned}
$$

where $[i]$ denotes the largest integer which does not exceed i. The k=1 term should be omitted from (5) for the first two values of i, and the k=2 term should be omitted for the last two values of i because the triangle functions do not overlap at the ends of the generating curve. Similarly, the $\ell=1$ term should be omitted for the first two values of j, and the $\ell=2$ term should be omitted for the last two values of j. The last three of equations (2) imply relations similar to (5).

The indices I and J of nested DO loops 60 and 59 are respectively i and j of (5). DO loop 61 accumulates G_1, G_2, and G_3 of (1-24)-(1-26) for I ≠ J and $.5G_1$, $.5G_2$, and $.5G_3$ for I = J in G1, G2, and G3. The $(Y1)_{ij}$, $(Y2)_{ij}$, $(Y3)_{ij}$ and $(Y4)_{ij}$ which appear in (5) and similar equations and which are given by (1-32)-(1-35) with the $\frac{\pi}{k^2 d_i \rho_i}$ terms missing and the i=j result halved are put in Y1, Y2, Y3, and Y4 just after DO loop 61. The indices K and L of nested DO loops 32 and 31 are respectively k and ℓ of (5). The Y(J1), Y(J2), Y(J3), and Y(J4) in DO loop 32 are respectively $(Y_n^{tt})_{(p+k)(q+\ell)}$, $Y_n^{\phi t})_{(p+k)(q+\ell)}$, $(Y_n^{t\phi})_{(p+k)(q+\ell)}$, and $(Y_n^{\phi\phi})_{(p+k)(q+\ell)}$.

DO loop 11 carries out (4) for

$$(p,q) = \begin{cases} (J,J) & J = 1 \\ (J,J),\ (J-1,J),\ \text{and}\ (J,J-1) & J > 1 \end{cases}$$

DO loop 11 also adds the contributions to (2) due to the $\frac{\pi}{k^2 d_i \rho_i}$ terms in (1-32) and (1-35). In DO loop 11, Y(KD1), Y(KD2), Y(KD3), and Y(KD4) are the diagonal (J,J) elements of Y_n^{tt}, $Y_n^{\phi t}$, $Y_n^{t\phi}$, and $Y_n^{\phi\phi}$ respectively. Similarly, Y(KU1), Y(KU2), Y(KU3), and Y(KU4) are the (J-1,J) elements while Y(KL1), Y(KL2), Y(KL3), and Y(KL4) are the (J, J-1) elements. The first R1 in DO loop 11 is the contribution to the diagonal elements of (2) due to the $\frac{\pi}{k^2 d_i \rho_i}$ terms. The second R1 in DO loop 11 is the contribution to the off diagonal elements due to the $\frac{\pi}{k^2 d_i \rho_i}$ terms.

Nested DO loops 13 and 14 use (1-39) for (i,j) = (I,J) to put $(Y_n^{tt})_{ij}$, $(Y_n^{\phi t})_{ij}$, $(Y_n^{t\phi})_{ij}$, and $(Y_n^{\phi\phi})_{ij}$ for $j \le i-2$ in Y(KL1), Y(KL2), Y(KL3), and Y(KL4) respectively.

B. LISTING OF THE SUBROUTINE YMAT

```
      SUBROUTINE YMAT(NN,NP,NPHI,RH,ZH,X,A,Y)
      COMPLEX U,Y(1600),G1,G2,G3,Y1,Y2,Y3,Y4
      DIMENSION RH(43),ZH(43),X(20),A(20),D(42),PD(42),CR(20),C1(20)
      DIMENSION C2(20),C3(20)
      COMMON RS(42),ZS(42),SV(42),CV(42),T(80)
      PI=3.141593
      DO 57 I=2,NP
      I2=I-1
      DR=RH(I)-RH(I2)
      DZ=ZH(I)-ZH(I2)
      D(I2)=SQRT(DR*DR+DZ*DZ)
      RS(I2)=.5*(RH(I)+RH(I2))
      ZS(I2)=.5*(ZH(I)+ZH(I2))
      SV(I2)=DR/D(I2)
      CV(I2)=DZ/D(I2)
      PD(I2)=PI/(D(I2)*RS(I2))
   57 CONTINUE
      NG=NP-1
      N2=NG-2
      N=N2/2
      J1=1
      J5=1
      DO 68 J=1,N
      D1=D(J1)
      D2=D(J1+1)
      D3=D(J1+2)
      D4=D(J1+3)
      DEL1=D1+D2
      DEL2=D3+D4
      T(J5)=.5*D1*D1/DEL1
      T(J5+1)=(D1+.5*D2)*D2/DEL1
      T(J5+2)=(D4+.5*D3)*D3/DEL2
      T(J5+3)=.5*D4*D4/DEL2
      J1=J1+2
      J5=J5+4
   68 CONTINUE
      PI2=.5*PI
      FN=NN
      DO 25 K=1,NPHI
      PH=PI2*(X(K)+1.)
      PHN=PH*FN
      SN=SIN(.5*PH)
      CR(K)=4.*SN*SN
      R1=PI2*A(K)
      C1(K)=.5*R1*CR(K)*COS(PHN)
      C2(K)=R1*COS(PH)*COS(PHN)
      C3(K)=R1*SIN(PH)*SIN(PHN)
   25 CONTINUE
      N2N=N2*N
      N4N=N2N*2
      DO 62 J=1,N4N
      Y(J)=0.
   62 CONTINUE
      U=(0.,1.)
      DO 59 J=1,NG
      L1=1
      L2=2
      IF(J.LE.2) L1=2
```

```
      IF(J.GT.N2) L2=1
      J8=J+1
      J9=J8/2
      JT=2*J9+J-6
      J5=(J9-3)*N2-2
      S1=1.
      DO 60 I=1,J
      I8=I+1
      I9=I8/2
      IT=2*I9+I-6
      J6=I9+J5
      RP=RS(J)-RS(I)
      ZP=ZS(J)-ZS(I)
      R2=RP*RP+ZP*ZP
      IF(I.NE.J) GO TO 41
      S1=.5
      R2=.C625*D(J)*C(J)
   41 R3=RS(I)*RS(J)
      G1=0.
      G2=0.
      G3=0.
      DO 61 K=1,NPHI
      R4=R2+R3*CR(K)
      R5=SCRT(R4)
      Y1=S1/(R4*R5)*(1.+U*R5)*(COS(R5)-U*SIN(R5))
      G1=C1(K)*Y1+G1
      G2=C2(K)*Y1+G2
      G3=C3(K)*Y1+G3
   61 CONTINUE
      G3=U*G3
      Y1=(RP*CV(J)-ZP*SV(J))*G2-RS(I)*CV(J)*G1
      Y2=(RS(J)*SV(I)*CV(J)-RS(I)*SV(J)*CV(I)-ZP*SV(I)*SV(J))*G3
      Y3=ZP*G3
      Y4=(RP*CV(I)-ZP*SV(I))*G2+RS(J)*CV(I)*G1
      K1=1
      K2=2
      IF(I.LE.2) K1=2
      IF(I.GT.N2) K2=1
      DO 31 L=L1,L2
      LT=JT+L+L
      J7=J6+L*N2
      DO 32 K=K1,K2
      KT=IT+K+K
      TT=T(LT)*T(KT)
      J1=J7+K
      J2=J1+N
      J3=J1+N2N
      J4=J3+N
      Y(J1)=TT*Y1+Y(J1)
      Y(J2)=TT*Y2+Y(J2)
      Y(J3)=TT*Y3+Y(J3)
      Y(J4)=TT*Y4+Y(J4)
   32 CONTINUE
   31 CONTINUE
   60 CONTINUE
   59 CONTINUE
      N2P=N2+1
      KD1=1
      J1=1
      J5=1
```

```
      DO 11 J=1,N
      KD2=KD1+N
      KD3=KD1+N2N
      KD4=KD3+N
      R2=T(J1+1)
      R3=T(J1+2)
      R4=T(J1+3)
      J6=J5+1
      R1=T(J1)*T(J1)*PD(J5)+R2*R2*PD(J6)+R3*R3*PD(J6+1)+R4*R4*PD(J6+2)
      G1=Y(KD1)-Y(KD4)
      Y(KD1)=R1+G1
      Y(KD2)=0.
      Y(KD3)=0.
      Y(KD4)=R1-G1
      IF(J.EQ.1) GO TO 22
      KU1=KD1-1
      KU2=KD2-1
      KU3=KD3-1
      KU4=KD4-1
      KL1=KD1-N2
      KL2=KD2-N2
      KL3=KD3-N2
      KL4=KD4-N2
      R1=T(J1)*T(J1-2)*PD(J5)+R2*T(J1-1)*PD(J6)
      G1=Y(KU1)-Y(KL4)
      G2=Y(KU4)-Y(KL1)
      Y(KU1)=R1+G1
      Y(KU2)=Y(KU2)-Y(KL2)
      Y(KU3)=Y(KU3)-Y(KL3)
      Y(KU4)=R1+G2
      Y(KL1)=R1-G2
      Y(KL2)=-Y(KL2)
      Y(KL3)=-Y(KU3)
      Y(KL4)=R1-G1
   22 KD1=KD1+N2P
      J1=J1+4
      J5=J5+2
   11 CONTINUE
      IF(N.LT.3) RETURN
      J2=N2
      DO 13 I=3,N
      J2=J2+N2
      J1=I-2
      KL1=I
      DO 14 J=1,J1
      KU1=J2+J
      KU2=KU1+N
      KU3=KU1+N2N
      KU4=KU3+N
      KL2=KL1+N
      KL3=KL1+N2N
      KL4=KL3+N
      Y(KL1)=-Y(KL4)
      Y(KL2)=-Y(KU2)
      Y(KL3)=-Y(KU3)
      Y(KL4)=-Y(KL1)
      KL1=KL1+N2
   14 CONTINUE
   13 CONTINUE
      RETURN
      END
```

III. THE SUBROUTINE ZMAT

A. Description

The subroutine ZMAT (NN, NP, NPHI, RH, ZH, X, A, Z) stores the matrix

$$[Z] = \begin{bmatrix} Z_n^{tt} & Z_n^{t\phi} \\ & \\ Z_n^{\phi t} & Z_n^{\phi\phi} \end{bmatrix} \tag{6}$$

appearing in the E-field matrix equation (1-57) by columns in Z. The elements of [Z] are given by (1-69)-(1-72). The input variables are defined in terms of variables appearing in reference [1] by

$$\mathrm{NN} = n$$

$$\mathrm{NP} = P$$

$$\mathrm{NPHI} = N_\phi$$

$$\mathrm{RH(i)} = k\rho_{\bar{i}}$$

$$\mathrm{ZH(i)} = kz_{\bar{i}}$$

$$\mathrm{X(k)} = x_k$$

$$\mathrm{A(k)} = A_k$$

In summary, n denotes $e^{jn\phi}$ dependence, $(\rho_{\bar{i}}, z_{\bar{i}})$, i=1,2,...,P are coordinates on the generating curve, the k which multiplies $\rho_{\bar{i}}$ and $z_{\bar{i}}$ is the propagation constant, and x_k and A_k are the abscissas and weights for the N_ϕ point Gaussian quadrature integration in ϕ.

Minimum allocations are given by

```
COMPLEX Z(4*N*N)
DIMENSION RH(NP), ZH(NP), X(NPHI), A(NPHI), D(NG),
   TP(4*N), CR(NPHI), C2(NPHI), C3(NPHI), C4(NPHI)
COMMON RS(NG), ZS(NG), SV(NG), CV(NG), T(4*N)
```

where

$$NG = NP-1$$

$$N = (NG-2)/2$$

The variables in common make the results of some intermediate calculations done in ZMAT available to the subroutine PLANE described in Section V.

DO loop 57 puts kd_i of (1-28) in D, $k\rho_i$ of (1-27) in RS, kz_i of (1-27) in ZS, $\sin v_i$ in SV, and $\cos v_i$ in CV. DO loop 68 puts (1-68) in TP and (1-29) in T. With regard to (1-62)-(1-65), DO loop 25 puts $4 \sin^2 (\frac{\phi_k}{2})$, $\frac{\pi}{2} A_k \cos \phi_k \cos (n\phi_k)$, $\frac{\pi}{2} A_k \sin \phi_k \sin (n\phi_k)$, and $\frac{\pi}{2} A_k \cos (n\phi_k)$ in CR, C2, C3, and C4 respectively where, as prescribed by (1-75),

$$\phi_k = \frac{\pi}{2} (x_k+1)$$

DO loop 62 initializes Z.

Equation (1-69) can be rewritten as

$$(Z_n^{tt})_{pq} = j \sum_{i=2p-1}^{2p+2} \sum_{j=2q-1}^{2q+2} \{T_{2p-2+i} T_{2q-2+j} (G_5 \sin v_i \sin v_j + G_4 \cos v_i \cos v_j) - T'_{2p-2+i} T'_{2q-2+j} G_4\} \tag{7}$$

where G_4 and G_5 are evaluated at $(\rho,z,\rho',z') = (\rho_i,z_i,\rho_j,z_j)$. From (7) and similar equations for $(Z_n^{\phi t})_{pq}$, $(Z_n^{t\phi})_{pq}$, and $(Z_n^{\phi\phi})_{pq}$, we obtain

$$\begin{aligned} ((Z_n^{tt})_{pq})_{ij} &= ((Z_n^{tt})_{qp})_{ji} \\ ((Z_n^{\phi t})_{pq})_{ij} &= -((Z_n^{t\phi})_{qp})_{ji} \\ ((Z_n^{\phi\phi})_{pq})_{ij} &= ((Z_n^{\phi\phi})_{qp})_{ji} \end{aligned} \tag{8}$$

where the additional subscripts i,j denote the contribution due to the ij^{th} term on the right-hand side of (7) and similar equations. Equation (8) implies that

$$(Z_n^{tt})_{pq} = ((Z_n^{tt})_{pq})_{i\underline{<}j} + ((Z_n^{tt})_{qp})_{i\underline{<}j}$$

$$(Z_n^{\phi t})_{pq} = ((Z_n^{\phi t})_{pq})_{i\underline{<}j} - ((Z_n^{t\phi})_{qp})_{i\underline{<}j}$$

$$(Z_n^{t\phi})_{pq} = ((Z_n^{t\phi})_{pq})_{i\underline{<}j} - ((Z_n^{\phi t})_{qp})_{i\underline{<}j} \tag{9}$$

$$(Z_n^{\phi\phi})_{pq} = ((Z_n^{\phi\phi})_{pq})_{i\underline{<}j} + ((Z_n^{\phi\phi})_{qp})_{i\underline{<}j}$$

where the subscript notation $i\underline{<}j$ denotes the sum of the contributions due to all terms for which $i<j$ plus half of the $i=j$ term on the right-hand side of (7) and similar equations. According to (9), it is sufficient to calculate only the $i\underline{<}j$ terms on the right-hand side of (7) and similar equations. Inspection of the limits of summation in (7) and similar equations shows that the first term on the right-hand side of (9) is zero if $p > q+1$ and that the second term is zero if $p < q-1$.

The ij^{th} term of (7) and similar equations contributes

$$\{T_{2(p+k-1)+i}T_{2(q+\ell-1)+j}(Z1) + T'_{2(p+k-1)+i}T'_{2(q+\ell-1)+j}(Z1A)\} \text{ to } (Z_n^{tt})_{(p+k)(q+\ell)},$$

$$T_{2(p+k-1)+i}\{T_{2(q+\ell-1)+j}(Z2) + T'_{2(q+\ell-1)+j}(Z2A)\} \text{ to } (Z_n^{\phi t})_{(p+k)(q+\ell)},$$

$$T_{2(q+\ell-1)+j}\{T_{2(p+k-1)+i}(Z3) + T'_{2(p+k-1)+i}(Z3A)\} \text{ to } (Z_n^{t\phi})_{(p+k)(q+\ell)}, \tag{10}$$

and

$$T_{2(p+k-1)+i}T_{2(q+\ell-1)+j}(Z4) \text{ to } (Z_n^{\phi\phi})_{(p+k)(q+\ell)}, \quad \left\{\begin{array}{l} k=1,2 \\ \ell=1,2 \end{array}\right.$$

where

$$
\begin{aligned}
Z1 &= j(G_5 \sin v_i \sin v_j + G_4 \cos v_i \cos v_j) \\
Z1A &= -jG_4 \\
Z2 &= -\ G_6 \sin v_j \\
Z2A &= -\frac{n}{k\rho_i} G_4 \\
Z3 &= G_6 \sin v_i \\
Z3A &= \frac{n}{k\rho_j} G_4 \\
Z4 &= j\left(G_5 - \frac{n^2}{k^2 \rho_i \rho_j} G_4\right)
\end{aligned}
\tag{11}
$$

In (10),

$$p = [(i+1)/2] - 2$$

$$q = [(j+1)/2] - 2$$

where $[i]$ denotes the largest integer which does not exceed i. The $k=1$ term should be omitted from (10) for the first two values of i, and the $k=2$ term should be omitted for the last two values of i because the triangle functions do not overlap at the ends of the generating curve. Similarly, the $\ell = 1$ term should be omitted for the first two values of j, and the $\ell = 2$ term should be omitted for the last two values of j.

The indices I and J of DO loops 60 and 59 are respectively i and j of (10). DO loop 61 accumulates G_4, G_5, and G_6 of (1-62)-(1-64) for $I \neq J$ and $.5G_4$, $.5G_5$, and $.5G_6$ for $I = J$ in G4, G5, and G6. The Z's defined by (11) for $I \neq J$ and these Z's divided by two for $I = J$ are put in Z1, G1, Z2, G2, Z3, G3, and Z4 just after DO loop 61. The indices K and L of nested DO loops 32 and 31 are respectively k and ℓ of (10). The Z(J1), Z(J2), Z(J3), and Z(J4) in DO loop 32 are

respectively $(Z_n^{tt})_{(p+k)(q+\ell)}$, $(Z_n^{\phi t})_{(p+k)(q+\ell)}$, $(Z_n^{t\phi})_{(p+k)(q+\ell)}$, and $(Z_n^{\phi\phi})_{(p+k)(q+\ell)}$.

DO loop 11 carries out (9) for

$$(p,q) = \begin{cases} (J,J) & J = 1 \\ (J,J),(J-1,J), \text{ and } (J,J-1) & J > 1 \end{cases}$$

In DO loop 11, Z(KD1), Z(KD2), Z(KD3), and Z(KD4) are the diagonal (J,J) elements of Z_n^{tt}, $Z_n^{\phi t}$, $Z_n^{t\phi}$, and $Z_n^{\phi\phi}$ respectively. Similarly, Z(KU1), Z(KU2), Z(KU3), and Z(KU4) are the (J-1,J) elements while Z(KL1), Z(KL2), Z(KL3), and Z(KL4) are the (J,J-1) elements.

Nested DO loops 13 and 14 use (1-76) for (i,j) = (I,J) to put $(Z_n^{tt})_{ij}$, $(Z_n^{\phi t})_{ij}$, $(Z_n^{t\phi})_{ij}$, and $(Z_n^{\phi\phi})_{ij}$ for $j \leq i-2$ in Z(KL1), Z(KL2), Z(KL3), and Z(KL4) respectively.

B. LISTING OF THE SUBROUTINE ZMAT

```fortran
      SUBROUTINE ZMAT(NN,NP,NPHI,RH,ZH,X,A,Z)
      COMPLEX U,Z(1600),G1,G2,G3,G4,G5,G6,Z1,Z2,Z3,Z4
      DIMENSION RH(43),ZH(43),X(20),A(20),D(42),TP(80),CR(20),C2(20)
      DIMENSION C3(20),C4(20)
      COMMON RS(42),ZS(42),SV(42),CV(42),T(80)
      PI=3.141593
      DO 57 I=2,NP
      I2=I-1
      DR=RH(I)-RH(I2)
      DZ=ZH(I)-ZH(I2)
      D(I2)=SQRT(DR*DR+DZ*DZ)
      RS(I2)=.5*(RH(I)+RH(I2))
      ZS(I2)=.5*(ZH(I)+ZH(I2))
      SV(I2)=DR/D(I2)
      CV(I2)=DZ/D(I2)
   57 CONTINUE
      NG=NP-1
      N2=NG-2
      N=N2/2
      J1=1
      J5=1
      DO 68 J=1,N
      D1=D(J1)
      D2=D(J1+1)
      D3=D(J1+2)
      D4=D(J1+3)
      DEL1=D1+D2
      DEL2=D3+D4
      J6=J5+1
      J7=J6+1
      J8=J7+1
      TP(J5)=D1/DEL1
      TP(J6)=D2/DEL1
      TP(J7)=-D3/DEL2
      TP(J8)=-D4/DEL2
      T(J5)=.5*D1*TP(J5)
      T(J6)=(D1+.5*D2)*TP(J6)
      T(J7)=-(D4+.5*D3)*TP(J7)
      T(J8)=-.5*D4*TP(J8)
      J1=J1+2
      J5=J5+4
   68 CONTINUE
      PI2=.5*PI
      FN=NN
      DO 25 K=1,NPHI
      PH=PI2*(X(K)+1.)
      PHN=PH*FN
      SN=SIN(.5*PH)
      CR(K)=4.*SN*SN
      R1=PI2*A(K)
      C2(K)=R1*COS(PH)*COS(PHN)
      C3(K)=R1*SIN(PH)*SIN(PHN)
      C4(K)=R1*COS(PHN)
   25 CONTINUE
      N2N=N2*N
      N4N=N2N*2
      DO 62 J=1,N4N
      Z(J)=0.
```

```
62 CONTINUE
   U=(0.,1.)
   DO 59 J=1,NG
   FJ=FN/RS(J)
   L1=1
   L2=2
   IF(J.LE.2) L1=2
   IF(J.GT.N2) L2=1
   J8=J+1
   J9=J8/2
   JT=2*J9+J-6
   J5=(J9-3)*N2-2
   S1=1.
   DO 60 I=1,J
   I8=I+1
   I9=I8/2
   IT=2*I9+I-6
   J6=I9+J5
   FI=FN/RS(I)
   RP=RS(J)-RS(I)
   ZP=ZS(J)-ZS(I)
   R2=RP*RP+ZP*ZP
   IF(I.NE.J) GO TO 41
   S1=.5
   R2=.0625*D(J)*D(J)
41 R3=RS(I)*RS(J)
   G4=0.
   G5=0.
   G6=0.
   DO 61 K=1,NPHI
   R4=R2+R3*CR(K)
   R5=SQRT(R4)
   Z1=S1/R5*(COS(R5)-U*SIN(R5))
   G4=C4(K)*Z1+G4
   G5=C2(K)*Z1+G5
   G6=C3(K)*Z1+G6
61 CONTINUE
   Z1=U*(SV(I)*SV(J)*G5+CV(I)*CV(J)*G4)
   G1=-U*G4
   Z2=-SV(J)*G6
   G2=-FI*G4
   Z3=SV(I)*G6
   G3=FJ*G4
   Z4=U*(G5-FI*G3)
   K1=1
   K2=2
   IF(I.LE.2) K1=2
   IF(I.GT.N2) K2=1
   DO 31 L=L1,L2
   LT=JT+L+L
   J7=J6+L*N2
   DO 32 K=K1,K2
   KT=IT+K+K
   TT=T(LT)*T(KT)
   J1=J7+K
   J2=J1+N
   J3=J1+N2N
   J4=J3+N
   Z(J1)=TT*Z1+TP(LT)*TP(KT)*G1+Z(J1)
   Z(J2)=TT*Z2+TP(LT)*T(KT)*G2+Z(J2)
```

```
      Z(J3)=TT*Z3+TP(KT)*T(LT)*G3+Z(J3)
      Z(J4)=TT*Z4+Z(J4)
   32 CONTINUE
   31 CONTINUE
   60 CONTINUE
   59 CONTINUE
      N2P=N2+1
      KD1=1
      DO 11 J=1,N
      KD2=KD1+N
      KD3=KD1+N2N
      KD4=KD3+N
      Z(KD1)=Z(KD1)+Z(KD1)
      Z(KD2)=Z(KD2)-Z(KD3)
      Z(KD3)=-Z(KD2)
      Z(KD4)=Z(KD4)+Z(KD4)
      IF(J.EQ.1) GO TO 22
      KU1=KD1-1
      KU2=KD2-1
      KU3=KD3-1
      KU4=KD4-1
      KL1=KD1-N2
      KL2=KD2-N2
      KL3=KD3-N2
      KL4=KD4-N2
      Z(KU1)=Z(KU1)+Z(KL1)
      Z(KU2)=Z(KU2)-Z(KL3)
      Z(KU3)=Z(KU3)-Z(KL2)
      Z(KU4)=Z(KU4)+Z(KL4)
      Z(KL1)=Z(KU1)
      Z(KL2)=-Z(KU3)
      Z(KL3)=-Z(KL2)
      Z(KL4)=Z(KU4)
   22 KD1=KD1+N2P
   11 CONTINUE
      IF(N.LT.3) RETURN
      J2=N2
      DO 13 I=3,N
      J2=J2+N2
      J1=I-2
      KL1=I
      DO 14 J=1,J1
      KU1=J2+J
      KU2=KU1+N
      KU3=KU1+N2N
      KU4=KU3+N
      KL2=KL1+N
      KL3=KL1+N2N
      KL4=KL3+N
      Z(KL1)=Z(KU1)
      Z(KL2)=-Z(KL3)
      Z(KL3)=-Z(KU2)
      Z(KL4)=Z(KU4)
      KL1=KL1+N2
   14 CONTINUE
   13 CONTINUE
      RETURN
      END
```

IV. THE SUBROUTINE YZ

A. Description:

The subroutine YZ(NN, NP, NPHI, RH, ZH, X, A, Y, Z) stores by columns [Y] of (1) appearing in the H-field matrix equation (1-17) and [Z] of (6) appearing in the E-field matrix equation (1-57) in Y and Z respectively. The subroutine YZ is the subroutines YMAT and ZMAT combined into one subroutine. If both [Y] and [Z] are required, some compile time and execution time can be saved by using YZ instead of YMAT and ZMAT. However, YZ requires that [Y] and [Z] be stored simultaneously whereas YMAT and ZMAT do not necessarily require separate storage locations for [Y] and [Z].

The input variables NN, NP, NPHI, RH, ZH, X, and A have the same meaning as in the subroutines YMAT and ZMAT.

Minimum allocations are given by

```
COMPLEX Y(4*N*N), Z(4*N*N)
DIMENSION RH(NP), ZH(NP), X(NPHI), A(NPHI), D(NG),
   PD(NG), TP(4*N), CR(NPHI), C1(NPHI), C2(NPHI),
   C3(NPHI), C4(NPHI)
COMMON RS(NG), ZS(NG), SV(NG), CV(NG), T(4*N)
```

where

$$NG = NP-1$$

$$N = (NG-2)/2$$

The variables in common make the results of some intermediate calculations done in YZ available to the subroutine PLANE described in Section V.

Because all variables used in YZ can be traced back to YMAT and ZMAT, anyone who has gone through YMAT and ZMAT should have no trouble with YZ.

B. LISTING OF THE SUBROUTINE YZ

```
      SUBROUTINE YZ(NN,NP,NPHI,RH,ZH,X,A,Y,Z)
      COMPLEXU,Y(1600),Z(1600),G1,G2,G3,G4,G5,G6,Y1,Y2,Y3,Y4,Z1,Z2,Z3,Z4
      DIMENSION RH(43),ZH(43),X(20),A(20),D(42),PD(42),TP(80),CR(20)
      DIMENSION C1(20),C2(20),C3(20),C4(20)
      COMMON RS(42),ZS(42),SV(42),CV(42),T(80)
      PI=3.141593
      DO 57 I=2,NP
      I2=I-1
      DR=RH(I)-RH(I2)
      DZ=ZH(I)-ZH(I2)
      D(I2)=SQRT(DR*DR+DZ*DZ)
      RS(I2)=.5*(RH(I)+RH(I2))
      ZS(I2)=.5*(ZH(I)+ZH(I2))
      SV(I2)=DR/D(I2)
      CV(I2)=DZ/D(I2)
      PD(I2)=PI/(D(I2)*RS(I2))
   57 CONTINUE
      NG=NP-1
      N2=NG-2
      N=N2/2
      J1=1
      J5=1
      DO 68 J=1,N
      D1=D(J1)
      D2=D(J1+1)
      D3=D(J1+2)
      D4=D(J1+3)
      DEL1=D1+D2
      DEL2=D3+D4
      J6=J5+1
      J7=J6+1
      J8=J7+1
      TP(J5)=D1/DEL1
      TP(J6)=D2/DEL1
      TP(J7)=-D3/DEL2
      TP(J8)=-D4/DEL2
      T(J5)=.5*D1*TP(J5)
      T(J6)=(D1+.5*D2)*TP(J6)
      T(J7)=-(D4+.5*D3)*TP(J7)
      T(J8)=-.5*D4*TP(J8)
      J1=J1+2
      J5=J5+4
   68 CONTINUE
      PI2=.5*PI
      FN=NN
      DO 25 K=1,NPHI
      PH=PI2*(X(K)+1.)
      PHN=PH*FN
      SN=SIN(.5*PH)
      CR(K)=4.*SN*SN
      R1=PI2*A(K)
      C1(K)=.5*R1*CR(K)*COS(PHN)
      C2(K)=R1*COS(PH)*COS(PHN)
      C3(K)=R1*SIN(PH)*SIN(PHN)
      C4(K)=R1*COS(PHN)
   25 CONTINUE
      N2N=N2*N
      N4N=N2N*2
```

```
      DO 62 J=1,N4N
      Y(J)=0.
      Z(J)=0.
   62 CONTINUE
      U=(0.,1.)
      DO 59 J=1,NG
      FJ=FN/RS(J)
      L1=1
      L2=2
      IF(J.LE.2) L1=2
      IF(J.GT.N2) L2=1
      J8=J+1
      J9=J8/2
      JT=2*J9+J-6
      J5=(J9-3)*N2-2
      S1=1.
      DO 60 I=1,J
      I8=I+1
      I9=I8/2
      IT=2*I9+I-6
      J6=I9+J5
      FI=FN/RS(I)
      RP=RS(J)-RS(I)
      ZP=ZS(J)-ZS(I)
      R2=RP*RP+ZP*ZP
      IF(I.NE.J) GO TO 41
      S1=.5
      R2=.C625*D(J)*C(J)
   41 R3=RS(I)*RS(J)
      G1=0.
      G2=0.
      G3=0.
      G4=0.
      G5=0.
      G6=0.
      DO 61 K=1,NPHI
      R4=R2+R3*CR(K)
      R5=SQRT(R4)
      Z1=S1/R5*(COS(R5)-U*SIN(R5))
      Y1=Z1*(1.+U*R5)/R4
      G1=C1(K)*Y1+G1
      G2=C2(K)*Y1+G2
      G3=C3(K)*Y1+G3
      G4=C4(K)*Z1+G4
      G5=C2(K)*Z1+G5
      G6=C3(K)*Z1+G6
   61 CONTINUE
      G3=U*G3
      Y1=(RP*CV(J)-ZP*SV(J))*G2-RS(I)*CV(J)*G1
      Y2=(RS(J)*SV(I)*CV(J)-RS(I)*SV(J)*CV(I)-ZP*SV(I)*SV(J))*G3
      Y3=ZP*G3
      Y4=(RP*CV(I)-ZP*SV(I))*G2+RS(J)*CV(I)*G1
      Z1=U*(SV(I)*SV(J)*G5+CV(I)*CV(J)*G4)
      G1=-U*G4
      Z2=-SV(J)*G6
      G2=-FI*G4
      Z3=SV(I)*G6
      G3=FJ*G4
      Z4=U*(G5-FI*G3)
      K1=1
```

```
      K2=2
      IF(I.LE.2) K1=2
      IF(I.GT.N2) K2=1
      DO 31 L=L1,L2
      LT=JT+L+L
      J7=J6+L*N2
      DO 32 K=K1,K2
      KT=IT+K+K
      TT=T(LT)*T(KT)
      J1=J7+K
      J2=J1+N
      J3=J1+N2N
      J4=J3+N
      Y(J1)=TT*Y1+Y(J1)
      Y(J2)=TT*Y2+Y(J2)
      Y(J3)=TT*Y3+Y(J3)
      Y(J4)=TT*Y4+Y(J4)
      Z(J1)=TT*Z1+TP(LT)*TP(KT)*G1+Z(J1)
      Z(J2)=TT*Z2+TP(LT)*T(KT)*G2+Z(J2)
      Z(J3)=TT*Z3+TP(KT)*T(LT)*G3+Z(J3)
      Z(J4)=TT*Z4+Z(J4)
   32 CONTINUE
   31 CONTINUE
   60 CONTINUE
   59 CONTINUE
      N2P=N2+1
      KD1=1
      J1=1
      J5=1
      DO 11 J=1,N
      KD2=KD1+N
      KD3=KD1+N2N
      KD4=KD3+N
      R2=T(J1+1)
      R3=T(J1+2)
      R4=T(J1+3)
      J6=J5+1
      R1=T(J1)*T(J1)*PD(J5)+R2*R2*PD(J6)+R3*R3*PD(J6+1)+R4*R4*PD(J6+2)
      G1=Y(KD1)-Y(KD4)
      Y(KD1)=R1+G1
      Y(KD2)=0.
      Y(KD3)=0.
      Y(KD4)=R1-G1
      Z(KD1)=Z(KD1)+Z(KD1)
      Z(KD2)=Z(KD2)-Z(KD3)
      Z(KD3)=-Z(KD2)
      Z(KD4)=Z(KD4)+Z(KD4)
      IF(J.EQ.1) GO TO 22
      KU1=KD1-1
      KU2=KD2-1
      KU3=KD3-1
      KU4=KD4-1
      KL1=KD1-N2
      KL2=KD2-N2
      KL3=KD3-N2
      KL4=KD4-N2
      R1=T(J1)*T(J1-2)*PD(J5)+R2*T(J1-1)*PD(J6)
      G1=Y(KU1)-Y(KL4)
      G2=Y(KU4)-Y(KL1)
      Y(KU1)=R1+G1
```

```
      Y(KU2)=Y(KU2)-Y(KL2)
      Y(KU3)=Y(KU3)-Y(KL3)
      Y(KU4)=R1+G2
      Y(KL1)=R1-G2
      Y(KL2)=-Y(KU2)
      Y(KL3)=-Y(KU3)
      Y(KL4)=R1-G1
      Z(KU1)=Z(KU1)+Z(KL1)
      Z(KU2)=Z(KU2)-Z(KL3)
      Z(KU3)=Z(KU3)-Z(KL2)
      Z(KU4)=Z(KU4)+Z(KL4)
      Z(KL1)=Z(KU1)
      Z(KL2)=-Z(KU3)
      Z(KL3)=-Z(KL2)
      Z(KL4)=Z(KU4)
   22 KD1=KD1+N2P
      J1=J1+4
      J5=J5+2
   11 CONTINUE
      IF(N.LT.3) RETURN
      J2=N2
      DC 13 I=3,N
      J2=J2+N2
      J1=I-2
      KL1=I
      DO 14 J=1,J1
      KU1=J2+J
      KU2=KU1+N
      KU3=KU1+N2N
      KL4=KU3+N
      KL2=KL1+N
      KL3=KL1+N2N
      KL4=KL3+N
      Y(KL1)=-Y(KU4)
      Y(KL2)=-Y(KU2)
      Y(KL3)=-Y(KU3)
      Y(KL4)=-Y(KU1)
      Z(KL1)=Z(KU1)
      Z(KL2)=-Z(KU3)
      Z(KL3)=-Z(KU2)
      Z(KL4)=Z(KU4)
      KL1=KL1+N2
   14 CCNTINUE
   13 CONTINUE
      RETURN
      END
```

V. THE SUBROUTINE PLANE

A. Description.

The subroutine PLANE(NN, N, NT, THR, R) stores by columns in R the matrix [R] given by

$$[R] = \begin{bmatrix} \vec{R}_n^{t\theta} & \vec{R}_n^{t\phi} \\ \\ \vec{R}_n^{\phi\theta} & \vec{R}_n^{\phi\phi} \end{bmatrix} \tag{12}$$

where the elements of the column vectors on the right-hand side of (12) are given by (1-95). The input variables NN and N are respectively n of (1-95) and the maximum value of i in (1-95). If NT = 1, then THR(1) is θ_r of (1-95) in radians. If NT > 1, then the measurement matrices [R] of (12) for θ_r = THR(I), I = 1,2,...NT are stored consecutively in R. The variables RS, ZS, SV, CV, and T appearing in the common statement early in the subroutine PLANE are input variables calculated by calling any one of the subroutines YMAT, ZMAT, or YZ beforehand. The calculated values of these variables depend only on the second, fourth,and fifth arguments (NP, RH, and ZH) of either YMAT, ZMAT, or YZ.

Minimum allocations are given by

```
COMPLEX R(4*N*NT)
DIMENSION THR(NT), BJ(M)
COMMON RS(NG), ZS(NG), SV(NG), CV(NG), T(4*N)
```

where

```
NG = 2*N + 2
```

and M is the largest of the values of M calculated by PLANE. The suggested allocation BJ(50) will work if the maximum circumference of the body of revolution is less than 26 wavelengths.

We rewrite (1-95) as

$$\begin{bmatrix} R_{np}^{t\theta} \\ R_{np}^{\phi\theta} \\ R_{np}^{t\phi} \\ R_{np}^{\phi\phi} \end{bmatrix} = \sum_{i=2p-1}^{2p+2} T_{i+2p-2} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} \tag{13}$$

where

$$\begin{aligned} R_1 &= \pi j^n(-2J_n \sin\theta_r \cos v + j(J_{n+1} - J_{n-1})\cos\theta_r \sin v)e^{jkz\cos\theta_r} \\ R_2 &= -\pi j^n(J_{n+1} + J_{n-1})\cos\theta_r\, e^{jkz\cos\theta_r} \\ R_3 &= \pi j^n(J_{n+1} + J_{n-1})\sin v\, e^{jkz\cos\theta_r} \\ R_4 &= \pi j^{n+1}(J_{n+1} - J_{n-1})e^{jkz\cos\theta_r} \end{aligned} \tag{14}$$

In (14), ρ, z, and v are to be evaluated at $t = t_i$. Equations (13) say that

$$T_{i+2p-2}R_1 \quad \text{contributes to} \quad R_{np}^{t\theta}\,,$$

$$T_{i+2p-2}R_2 \quad \text{contributes to} \quad R_{np}^{\phi\theta},$$

$$T_{i+2p-2}R_3 \quad \text{contributes to} \quad R_{np}^{t\phi}\,, \text{ and}$$

$$T_{i+2p-2}R_4 \quad \text{contributes to} \quad R_{np}^{\phi\phi}$$

for

$$p = [\frac{i+1}{2}] - 1$$

and

$$p = [\frac{i+1}{2}]$$

Because the triangle functions do not overlap at the ends of the generating curve, the first value of p must be discarded if i is either 1 or 2 and the

second value of p must be discarded if i is either 2*N+1 or 2*N+2.

In DO loop 12, θ_r of (14) is THR(L). DO loop 13 evaluates (14) at $t = t_I$. The logic in DO loop 13 prior to statement 24 puts the Bessel functions $J_{n-1}(x)$, $J_n(x)$, and $J_{n+1}(x)$ in BJ1, BJ2, and BJ3 respectively where

$$x = k\rho_I \sin \theta_r$$

Choosing M so large that the magnitude of $J_{M-1}(x)$ is around 10^{-8}, we start with

$$J_{M-1}(x) = 0$$

$$J_{M-2}(x) = 1$$

and use the recurrence relation

$$J_{n-1}(x) = \frac{2n}{x} J_n(x) - J_{n+1}(x) \tag{15}$$

as given by (9.1.27) on page 361 of reference [2] to calculate $J_n(x)$ for n = M-3, M-4, ... 0 and then normalize $J_n(x)$ according to

$$1 = J_0(x) + 2J_2(x) + 2J_4(x) + 2J_6(x) + \ldots \tag{16}$$

as given by (9.1.46) on page 361 of reference [2]. Statement 24 and the 6 statements following it put R_1, R_2, R_3, and R_4 of (14) in R1, R2, R3 and R4 respectively. DO loop 20 adds the contributions to (13) for i = I and

$$p = [\frac{I + 1}{2}] + K-2$$

[2] M. Abramowitz and I. A. Stegun, "Handbook of Mathematical Functions," U. S. Govt. Printing Office, Washington, D. C. (Natl. Bur. Std. U. S. Applied Math. Ser. 55), 1964.

B. LISTING CF THE SUBROUTINE PLANE

```
      SUBRCUTINE PLANE(NN,N,NT,THR,R)
      COMPLEX R(240),U,U1,U2,R1,R2,R3,R4
      DIMENSION THR(3),BJ(50)
      CCMMCN RS(42),ZS(42),SV(42),CV(42),T(80)
      N2=2*N
      NG=N2+2
      U=(0.,1.)
      U1=3.141593*U**NN
      JR=4*N*NT
      DO 22 J=1,JR
      R(J)=0.
   22 CONTINUE
      J5=-2
      DO 12 L=1,NT
      CS=CCS(THR(L))
      SN=2.*SIN(THR(L))
      DO 13 I=1,NG
      X=.25*RS(I)*SN
      IF(X.LE..5E-7) GO TO 18
      M=2.8*X+13.-2./X
      IF(X.LT..5) M=10.8+ALOG10(X)
      IF(M.GE.(NN+2)) GC TC 19
   18 BJ1=C.
      BJ2=0.
      BJ3=C.
      IF(NN.EQ.1) BJ1=1.
      IF(NN.EQ.0) BJ2=1.
      GO TO 24
   19 BJ(M)=0.
      JM=M-1
      BJ(JM)=1.
      DC 14 J=3,M
      JM=JM-1
      BJ(JM)=JM/X*BJ(JM+1)-BJ(JM+2)
   14 CCNTINUE
      S=0.
      DO 15 J=3,M,2
      S=S+BJ(J)
   15 CONTINUE
      S=BJ(1)+2.*S
      BJ2=BJ(NN+1)/S
      BJ3=BJ(NN+2)/S
      BJ1=-BJ3
      IF(NN.GT.0) BJ1=BJ(NN)/S
   24 ARG=ZS(I)*CS
      U2=U1*(COS(ARG)+U*SIN(ARG))
      R4=(BJ3-BJ1)*U*U2
      R2=(BJ3+BJ1)*U2
      R1=-BJ2*CV(I)*SN*U2+CS*SV(I)*R4
      R3=SV(I)*R2
      R2=-CS*R2
      I9=(I+1)/2
      IT=2*I9+I-6
      J7=I9+J5
      K1=1
      K2=2
      IF(I.LE.2) K1=2
      IF(I.GT.N2) K2=1
```

```
      DO 20 K=K1,K2
      TT=T(IT+K+K)
      J1=J7+K
      J2=J1+N
      J3=J2+N
      J4=J3+N
      R(J1)=TT*R1+R(J1)
      R(J2)=TT*R2+R(J2)
      R(J3)=TT*R3+R(J3)
      R(J4)=TT*R4+R(J4)
   20 CONTINUE
   13 CONTINUE
      J5=J4-2
   12 CONTINUE
      RETURN
      END
```

VI. THE SUBROUTINES DECOMP AND SOLVE

A. Description:

The subroutines DECOMP(N,IPS,UL) and SOLVE(N,IPS,UL,B,X) solve a system of N linear equations in N unknowns. These subroutines will be used in Section VII to solve the H-field matrix equations (1-17), the E-field matrix equations (1-57) and the combined field matrix equations (1-88). The input to DECOMP consists of N and the N by N matrix of coefficients on the left-hand side of the matrix equation stored by columns in UL. The output from DECOMP is IPS and UL. This output is fed into SOLVE. The rest of the input to SOLVE consists of N and the column of coefficients on the right-hand side of the matrix equation stored in B. SOLVE puts the solution to the matrix equation in X.

Minimum allocations are given by

```
COMPLEX UL(N*N)
DIMENSION SCL(N), IPS(N)
```

in DECOMP and by

```
COMPLEX UL(N*N), B(N), X(N)
DIMENSION IPS(N)
```

in SOLVE.

A description of DECOMP and SOLVE is on pages 46-49 of reference [3].

---

[3] J. R. Mautz and R. F. Harrington, "Transmission from a Rectangular Waveguide into Half Space Through a Rectangular Aperture," Interim Technical Report No. 12, Rome Air Development Center, Griffiss Air Force Base, New York, August 1976.

B. LISTING CF THE SUBROUTINE DECOMP

```
      SUBROUTINE CECCMP(N,IPS,UL)
      COMPLEX UL(160C),PIVOT,EM
      DIMENSION SCL(40),IPS(40)
      DO 5 I=1,N
      IPS(I)=I
      RN=0.
      J1=I
      DO 2 J=1,N
      ULM=ABS(REAL(UL(J1)))+ABS(AIMAG(UL(J1)))
      J1=J1+N
      IF(RN-ULM) 1,2,2
    1 RN=ULM
    2 CONTINUE
      SCL(I)=1 /RN
    5 CONTINUE
      NM1=N-1
      K2=0
      DO 17 K=1,NM1
      BIG=0.
      DO 11 I=K,N
      IP=IPS(I)
      IPK=IP+K2
      SIZE=(ABS(REAL(UL(IPK)))+ABS(AIMAG(UL(IPK))))*SCL(IP)
      IF(SIZE-BIG) 11,11,10
   10 BIG=SIZE
      IPV=I
   11 CONTINUE
      IF(IPV-K) 14,15,14
   14 J=IPS(K)
      IPS(K)=IPS(IPV)
      IPS(IPV)=J
   15 KPP=IPS(K)+K2
      PIVOT=UL(KPP)
      KP1=K+1
      DO 16 I=KP1,N
      KP=KPP
      IP=IPS(I)+K2
      EM=-UL(IP)/PIVOT
   18 UL(IP)=-EM
      DO 16 J=KP1,N
      IP=IP+N
      KP=KP+N
      UL(IP)=UL(IP)+EM*UL(KP)
   16 CONTINUE
      K2=K2+N
   17 CONTINUE
      RETURN
      END
C
C     LISTING OF THE SUBROUTINE SCLVE
C
      SUBROUTINE SCLVE(N,IPS,UL,B,X)
      CCMPLEX UL(16CC),B(40),X(40),SUM
      DIMENSION IPS(40)
      NP1=N+1
      IP=IPS(1)
      X(1)=B(IP)
      DO 2 I=2,N
```

```
      IP=IPS(I)
      IPB=IP
      IM1=I-1
      SUM=C.
      DO 1 J=1,IM1
      SUM=SUM+LL(IP)*X(J)
    1 IP=IP+N
    2 X(I)=B(IPB)-SUM
      K2=N*(N-1)
      IP=IPS(N)+K2
      X(N)=X(N)/UL(IP)
      DO 4 IBACK=2,N
      I=NP1-IBACK
      K2=K2-N
      IPI=IPS(I)+K2
      IP1=I+1
      SUM=C.
      IP=IPI
      DO 3 J=IP1,N
      IP=IP+N
    3 SUM=SUM+UL(IP)*X(J)
    4 X(I)=(X(I)-SUM)/UL(IPI)
      RETURN
      END
```

VII. THE MAIN PROGRAM

A. Description:

The main program uses all the subroutines listed in Sections II-VI to evaluate the surface currents (1-109) at one specified value of ϕ for both θ and ϕ transmitter polarizations and to evaluate the scattering cross section (1-111) at one specified value of ϕ_r for the four different combinations of transmitter and receiver polarizations. The main program is not general enough to meet every user's needs. Its main purpose is to give an example of how these subroutines can be used and to provide the user with sample output from them so that he can be sure that they are working properly.

Input data is read from punched cards according to

```
      READ(1,51) NM, NP, NPHI
51    FORMAT (3I3)
      READ(1,50) BK, TT, P, TR, PR, ALP
50    FORMAT (5E14.7)
      READ(1,46)(RH(I), I = 1, NP)
      READ(1,46)(ZH(I), I = 1, NP)
46    FORMAT (10F8.4)
      READ(1,50)(X(K), K = 1, NPHI)
      READ(1,50)(A(K), K = 1, NPHI)
```

The above input variables are defined by

NM = one plus the maximum value of n in (1-109) and (1-110)

NP = P

NPHI = N_ϕ

BK = k

TT = θ_t

P = ϕ

TR = θ_r

PR = ϕ_r

ALP = α

RH(i) = $k\bar{\rho}_i$

ZH(i) = $k\bar{z}_i$

X(k) = x_k

A(k) = A_k

where all variables on the right-hand sides of the above equations appear in reference [1]. According to reference [1], n denotes $e^{jn\phi}$ dependence, $(\bar{\rho}_i, \bar{z}_i)$, i=1,2,...P are coordinates on the generating curve, k is the propagation constant, θ_t is the colatitude of the transmitter, and ϕ is

the longitude at which the surface current is evaluated. The angles θ_r and ϕ_r are the colatitude and longitude of the receiver. The real constant α is the relative weight of the E-field integral equation in the combined field formulation. Finally, x_k and A_k are the abscissas and weights for the N_ϕ point Gaussian quadrature integration in ϕ. All angles read into the main program are in degrees.

Minimum allocations are given by

```
COMPLEX TJ(6*N), PJ(6*N), RT(4*N), RR(4*N),
   Y(4*N*N), Z(4*N*N), B(2*N), C(2*N)
DIMENSION RH(NP), ZH(NP), X(NPHI), A(NPHI),
   THT(1), THR(1), R2(N), IPS(2*N)
```

where

$$N = (NP - 3)/2$$

The surface current printed by the main program is not (1-109) per se, but (1-109) divided by the propagation constant k. The heading above the surface current in the printed output denotes real and imaginary parts and t and ϕ components. As further identification, NHEC=1 denotes the H-field solution, NHEC=2 denotes the E-field solution, NHEC=3 denotes the combined field solution, KT = 1 denotes a θ polarized transmitter, and KT = 2 denotes a ϕ polarized transmitter. The scattering cross section per square wavelength (1-111) is labeled SIGMA/(LAMBDA)**2 and further identified by the parameters NHEC, KT, and KR where KR = 1 denotes a θ polarized receiver and KR = 2 denotes a ϕ polarized receiver.

The index K of DO loop 41 obtains the n=K-1 term in (1-109) and (1-110). In DO loop 58, NHEC=1 obtains the H-field solution, NHEC=2 obtains the E-field solution, and NHEC=3 obtains the combined field solution. In DO loop 27, KT=1 denotes a θ polarized transmitter and KT=2 denotes a ϕ polarized transmitter. In DO loop 16, KR=1 denotes a θ polarized receiver and KR=2 denotes a ϕ polarized receiver. The user can omit computations which are of no interest to him by changing the statements which introduce DO loops 41, 58, 27, and 16 so as to restrict the indices of these DO loops. The logic in DO loop 58 before statement 59 puts the square matrix on the left-hand side of either the H-field matrix equation (1-17), the E-field matrix equation (1-57), or the combined

field matrix equation (1-88) in Y. The logic in DO loop 27 before statement 53 uses (1-99), (1-100) and (1-104) to put the excitation column vector on the right-hand side of either the H-field, the E-field, or the combined field matrix equation in B. Statement 53 puts the solution of the matrix equation in C. If KT=1, the logic in DO loop 13 adds the n=K-1 term of the t and ϕ components of the numerator of the first of equations (1-109) to TJ and PJ respectively. DO loop 15 adds the n=K-1 term of the t and ϕ components of the numerator of the second of equations (1-109) to TJ and PJ respectively. DO loop 16 adds the n=K-1 term of

$$\frac{4\pi r_r e^{jkr_r}}{-j\eta} E^s_{pq}(\underset{\sim}{J})$$

to E(KE) where $E^s_{pq}(\underset{\sim}{J})$ is given by (1-110). Here, p is either θ or ϕ, and q is either θ or ϕ.

Nested DO loops 28, 29, and 30 print out both (1-109) divided by the propagation constant k and (1-111). The indices NHEC, KT, and KR of these DO loops have the same meaning as in nested DO loops 58, 27, and 16.

The sample input and output data is for oblique incidence on a conducting sphere of radius 0.2 wavelengths. Only the n=0 and n=1 terms in (1-109) and (1-110) are considered. A few more terms may be needed to obtain reasonable accuracy.

B. LISTING OF THE MAIN PROGRAM

```
//PGM JOB (XXXX,XXXX,2,2),'MAUTZ,JOE',REGION=200K
// EXEC WATFIV
//GO.SYSIN DD *
$JOB              MAUTZ,TIME=2,PAGES=40
C     THIS PROGRAM RAN FOR 45 SECONDS ON THE IBM 370/155.
C     SUBROUTINES YMAT, ZMAT, YZ, PLANE, DECOMP, AND SOLVE ARE CALLED.
      COMPLEX U,TJ(120),PJ(120),E(12),SN,SNR,RT(240),RR(240),Y(1600)
      COMPLEX Z(1600),B(40),C(40),U1,CONJG
      DIMENSION RH(43),ZH(43),X(20),A(20),THT(3),THR(3),R2(20),IPS(40)
      READ(1,51) NM,NP,NPHI
   51 FORMAT(3I3)
      READ(1,50) BK,TT,P,TR,PR,ALP
   50 FORMAT(5E14.7)
      READ(1,46)(RH(I),I=1,NP)
      READ(1,46)(ZH(I),I=1,NP)
   46 FORMAT(10F8.4)
      READ(1,50)(X(K),K=1,NPHI)
      READ(1,50)(A(K),K=1,NPHI)
      WRITE(3,49) NM,NP,NPHI
   49 FORMAT('  NM NP NPHI'/1X,2I3,I4)
      WRITE(3,48) BK,TT,P,TR,PR,ALP
   48 FORMAT(7X,'BK',12X,'TT',13X,'P',12X,'TR',12X,'PR'/1X,5E14.7/7X,'AL
     1P'/1X,E14.7)
      WRITE(3,45)(RH(I),I=1,NP)
   45 FORMAT(' RH'/(1X,10F8.4))
      WRITE(3,44)(ZH(I),I=1,NP)
   44 FORMAT(' ZH'/(1X,10F8.4))
      WRITE(3,47)(X(K),K=1,NPHI)
   47 FORMAT(' X'/(1X,5E14.7))
      WRITE(3,43)(A(K),K=1,NPHI)
   43 FORMAT(' A'/(1X,5E14.7))
      N2=NP-3
      N=N2/2
      N3=N2+N
      N4=N2*N2
      N6=6*N
      U=(0.,1.)
      PI=3.141593
      P4=.0625/PI**3
      P8=PI/180.
      THT(1)=TT*P8
      P=P*P8
      THR(1)=TR*P8
      PR=PR*P8
      DO 42 J=1,NP
      RH(J)=BK*RH(J)
      ZH(J)=BK*ZH(J)
   42 CONTINUE
      DO 17 J=1,N
      R2(J)=1./RH(2*J+1)
   17 CONTINUE
      DO 54 J=1,N6
      TJ(J)=0.
      PJ(J)=0.
   54 CONTINUE
      DO 55 J=1,12
      E(J)=0.
   55 CONTINUE
```

```
      WRITE(3,9)
    9 FORMAT('OSAMPLE CUTPUT FRCM SUBRCUTINES')
      DO 41 K=1,NM
      NN=K-1
      PN=NN*P
      CS=COS(PN)
      SN=2.*SIN(PN)*U
      PN=NN*PR
      CSR=COS(PN)
      SNR=2.*SIN(PN)*U
      IF(NN.EQ.0) GO TO 56
      CS=2.*CS
      CSR=2.*CSR
   56 LANE=0
      DO 58 NHEC=1,3
      GO TO (61,62,63),NHEC
   61 CALL YMAT(NN,NP,NPHI,RH,ZH,X,A,Y)
      WRITE(3,8) Y(1),Y(2)
    8 FORMAT(1X,4E14.7)
      GO TO 59
   62 CALL ZMAT(NN,NP,NPHI,RH,ZH,X,A,Y)
      WRITE(3,8) Y(1),Y(2)
      GO TO 59
   63 CALL YZ(NN,NP,NPHI,RH,ZH,X,A,Y,Z)
      WRITE(3,8) Y(1),Y(2),Z(1),Z(2)
      DO 66 J=1,N4
      Y(J)=Y(J)+ALP*Z(J)
   66 CONTINUE
   59 CALL DECOMP(N2,IPS,Y)
      WRITE(3,8) Y(1),Y(2)
      IF(LANE.NE.C) GO TO 57
      LANE=1
      CALL PLANE(NN,N,1,THT,RT)
      CALL PLANE(NN,N,1,THR,RR)
      WRITE(3,8) RT(1),RT(2),RR(1),RR(2)
   57 DO 27 KT=1,2
      L=2*(NHEC-1)+KT
      GO TO (31,32,33,34,35,36),L
   31 DO 21 J=1,N
      B(J)=-RT(J+N3)
      B(J+N)=-RT(J+N2)
   21 CONTINUE
      GO TO 53
   32 DO 22 J=1,N
      JN=J+N
      B(J)=-RT(JN)
      B(JN)=-RT(J)
   22 CONTINUE
      GO TO 53
   33 DO 23 J=1,N
      B(J)=RT(J)
      JN=J+N
      B(JN)=-RT(JN)
   23 CONTINUE
      GO TO 53
   34 DO 24 J=1,N
      B(J)=-RT(J+N2)
      B(J+N)=RT(J+N3)
   24 CONTINUE
      GO TO 53
```

```
35 DO 25 J=1,N
   B(J)=-RT(J+N3)+ALP*RT(J)
   JN=J+N
   B(JN)=-RT(J+N2)-ALP*RT(JN)
25 CONTINUE
   GO TO 53
36 DO 26 J=1,N
   JN=J+N
   B(J)=-RT(JN)-ALP*RT(J+N2)
   B(JN)=-RT(J)+ALP*RT(J+N3)
26 CONTINUE
53 CALL SOLVE(N2,IPS,Y,B,C)
   WRITE(3,8) C(1),C(2)
   J1=(L-1)*N
   GO TO (11,12),KT
11 DO 13 J=1,N
   J2=J+J1
   TJ(J2)=TJ(J2)+C(J)*CS
   PJ(J2)=PJ(J2)+C(J+N)*SN
13 CONTINUE
   GO TO 14
12 DO 15 J=1,N
   J2=J+J1
   TJ(J2)=TJ(J2)+C(J)*SN
   PJ(J2)=PJ(J2)+C(J+N)*CS
15 CONTINUE
14 DO 16 KR=1,2
   L=(KT-1)*2+KR
   KE=4*(NHEC-1)+L
   U1=0.
   GO TO (71,72,73,74),L
71 DO 75 J=1,N2
   U1=U1+RR(J)*C(J)
75 CONTINUE
   E(KE)=E(KE)+U1*CSR
   GO TO 16
72 DO 76 J=1,N2
   U1=U1+RR(J+N2)*C(J)
76 CONTINUE
   E(KE)=E(KE)+U1*SNR
   GO TO 16
73 DO 77 J=1,N2
   U1=U1+RR(J)*C(J)
77 CONTINUE
   E(KE)=E(KE)+U1*SNR
   GO TO 16
74 DO 78 J=1,N2
   U1=U1+RR(J+N2)*C(J)
78 CONTINUE
   E(KE)=E(KE)+U1*CSR
16 CONTINUE
27 CONTINUE
58 CONTINUE
41 CONTINUE
   DO 28 NHEC=1,3
   DO 29 KT=1,2
   WRITE(3,18) NHEC,KT
18 FORMAT('ONHEC=',I3,', KT=',I3)
   WRITE(3,19)
19 FORMAT('   REAL JT    IMAG JT    REAL JP    IMAG JP')
```

```
      J1=N*(2*NHEC+KT-3)
      DO 37 J=1,N
      J2=J+J1
      TJ(J2)=TJ(J2)*R2(J)
      PJ(J2)=PJ(J2)*R2(J)
      WRITE(3,38) TJ(J2),PJ(J2)
   38 FORMAT(1X,4E11.4)
   37 CONTINUE
      DO 30 KR=1,2
      J1=4*NHEC+2*KT+KR-6
      SIG=P4*E(J1)*CONJG(E(J1))
      WRITE(3,10) NHEC,KT,KR,SIG
   10 FORMAT(' NHEC=',I3,', KT=',I3,', KR=',I3,', SIGMA/(LAMBDA)**2=',E1
     11.4)
   30 CONTINUE
   29 CONTINUE
   28 CONTINUE
      STOP
      END
$DATA
  2 21 20
 0.1256637E+01 0.3000000E+02 0.4500000E+02 0.6000000E+02 0.2000000E+02
 0.2000000E+00
  0.0000   0.1564   0.3090   0.4540   0.5878   0.7071   0.8090   0.8910   0.9511   0.9877
  1.0000   0.9877   0.9511   0.8910   0.8090   0.7071   0.5878   0.4540   0.3090   0.1564
  0.0000
 -1.0000  -0.9877  -0.9511  -0.8910  -0.8090  -0.7071  -0.5878  -0.4540  -0.3090  -0.1564
  0.0000   0.1564   0.3090   0.4540   0.5878   0.7071   0.8090   0.8910   0.9511   0.9877
  1.0000
-0.9931286E+00-0.9639719E+00-0.9122344E+00-0.8391170E+00-0.7463319E+00
-0.6360537E+00-0.5108670E+00-0.3737061E+00-0.2277859E+00-0.7652652E-01
 0.7652652E-01 0.2277859E+00 0.3737061E+00 0.5108670E+00 0.6360537E+00
 0.7463319E+00 0.8391170E+00 0.9122344E+00 0.9639719E+00 0.9931286E+00
 0.1761401E-01 0.4060143E-01 0.6267205E-01 0.8327674E-01 0.1019301E+00
 0.1181945E+00 0.1316886E+00 0.1420961E+00 0.1491730E+00 0.1527534E+00
 0.1527534E+00 0.1491730E+00 0.1420961E+00 0.1316886E+00 0.1181945E+00
 0.1019301E+00 0.8327674E-01 0.6267205E-01 0.4060143E-01 0.1761401E-01
$STOP
/*
//
```

```
PRINTED OUTPUT
 NM NP NPHI
  2 21  20
      BK                TT                P                 TR                PR
 0.1256637E+01 0.3000000E+02 0.4500000E+02 0.6000000E+02 0.2000000E+02
      ALP
 0.2000000E+00
RH
  0.0000   0.1564   0.3090   0.4540   0.5878   0.7071   0.8090   0.8910   0.9511   0.9877
  1.0000   0.9877   0.9511   0.8910   0.8090   0.7071   0.5878   0.4540   0.3090   0.1564
  0.0000
ZH
 -1.0000  -0.9877  -0.9511  -0.8910  -0.8090  -0.7071  -0.5878  -0.4540  -0.3090  -0.1564
  0.0000   0.1564   0.3090   0.4540   0.5878   0.7071   0.8090   0.8910   0.9511   0.9877
  1.0000
X
-0.9931286E+00-0.9639719E+00-0.9122344E+00-0.8391170E+00-0.7463319E+00
-0.6360537E+00-0.5108670E+00-0.3737061E+00-0.2277859E+00-0.7652652E-01
 0.7652652E-01 0.2277859E+00 0.3737061E+00 0.5108670E+00 0.6360537E+00
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 0.1181945E+00 0.1316886E+C0 0.142C961E+00 0.1491730E+00 0.1527534E+00
 0.1527534E+00 0.1491730E+00 0.1420961E+00 0.1316886E+C0 0.1181945E+00
 C.1019301E+00 0.8327675E-01 0.6267208E-01 0.4060143E-01 0.1761401E-01

SAMPLE CUTPUT FROM SUBROUTINES
 0.2096914E+01 0.1786422E-01 0.2827516E+00 0.3195152E-01
 C.2096914E+01 0.1786422E-01 0.1349617E+00 0.1408762E-01
-C.4120092E-01 0.4162729E+00-0.2257579E+00 0.7185605E+C0
-0.4243653E+00 0.5008702E+00-C.8416800E+00 0.7853127E+00
-0.1092077E+00-0.1034844E-01-0.5291492E+00-0.1006212E+00
 C.0000000E+00 0.CCCC000E+00 0.0000000E+00 0.0000000E+00
 C.3091431E-01-0.1C75599E+02 0.5109501E-01 0.6999979E+00
 C.3091431E-01-0.1C75599E+02-0.6506562E-01 0.4937384E-C2
-0.1230594E+00-0.1179976E-01-0.5162898E+00-0.1013399E+00
 0.0000000E+00 0.CC0C000E+00 C.000C000E+00 0.0C00000E+00
 C.2096915E+01 0.1786422E-01 0.2827514E+00 0.3195150E-01
 C.3091431E-C1-0.1C75599E+02 0.5109501E-01 0.6999979E+00
 0.2103098E+01-0.2133332E+01 0.2778168E-01 0.1099419E+C0
-0.1184121E+00-0.4237223E-02-0.5216529E+00-0.1052166E+00
 0.0000000E+00 C.CCC0000E+00 0.C00C000E+00 0.0C00000E+00
 0.2437500E+01-0.1057037E-01 0.5140911E+00-0.2176931E-01
 C.2437500E+01-0.1057037E-01 0.2109438E+00-0.8016225E-02
 C.4807118E+00-0.8731053E+00 0.4201249E+00-C.7168542E+C0
 C.3939573E+00-0.4160154E+00 0.1906640E+00-0.5476063E+00
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 C.8862776E-01 0.2205508E+00 0.5910277E-02 0.3503525E+00
 C.1875597E+00-0.1473804E+00 0.3282781E+00-0.8860910E-01
 0.2437500E+01-0.1057036E-01 0.5140910E+00-0.2176930E-01
 C.1408945E+0C-0.5389362E+01 0.1184365E+00 0.1649075E+01
 C.2465678E+01-0.1C88442E+01 0.1363794E+00 0.1851363E+00
 C.7738441E-01 0.2207815E+00 0.5051553E-02 0.3520448E+00
 C.1917793E+00-0.1352912E+C0 0.3317596E+00-0.8850223E-01

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-C.7018E+00 0.5397E+C0-0.2263E+00-C.5920E+00
-0.1187E+01 0.1405E+CC-0.2110E+00-0.3438E+00
-C.1440E+01-0.3538E+00-0.3408E+00-0.1353E+00
-C.1410E+01-0.8C99E+C0-0.5889E+00-C.1008E+00
-0.1199E+01-0.1133E+01-0.8275E+00-C.2960E+00
-C.9433E+00-0.13C8E+C1-0.9319E+00-C.5589E+00
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-C.2673E+00 0.4801E+00 0.8414E+00 0.6892E-01
-0.7001E+00 0.2035E+CC 0.1144E+01 0.2235E+00
-0.1004E+01-0.1645E+C0 0.1288E+01 0.6264E+00
```

```
-0.1116E+01-0.5484E+C0 0.1190E+01 0.1076E+01
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-C.9129E+00-0.1C75E+C1 0.7419E+00 C.1496E+01
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NHEC=  2, KT=  1
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-C.6876E+0C 0.5336E+C0-0.1304E+00-C.5761E+00
-0.1194E+01 0.1322E+C0-0.2399E+00-C.3514E+00
-C.1434E+01-0.3588E+C0-0.3084E+00-0.?024E-01
-C.1408E+01-0.8132E+C0-0.6047E+00-0.8353E-01
-C.1191E+01-0.1136E+C1-0.8651E+00-C.2606E+00
-C.9372E+0C-0.1312E+C1-0.9411E+00-C.6777E+00
-0.7382E+00-0.1390E+01-0.9732E+00-0.1011E+01
-C.6682E+00-0.1400E+01-0.7495E+00-0.1341E+01
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  REAL JT     IMAG JT     REAL JP     IMAG JP
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 0.1696E+00 0.6285E+00 0.4836E+00 0.1829E+00
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-C.6966E+00 0.5316E+C0-0.2239E+00-C.5899E+00
-0.1195E+01 0.1346E+C0-0.2115E+00-0.3417E+00
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-0.2716E+00 0.4793E+00 0.8441E+00 0.6579E-01
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-C.1002E+01-0.1654E+C0 0.1287E+01 0.6279E+00
-C.1111E+01-0.5490E+CC 0.1186E+01 0.1077E+01
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VIII. EXAMPLES AND DISCUSSION

Some examples of computations obtained from the computer program are given in the previous report [1]. We here give some additional examples to illustrate the use of the program.

As discussed in [1], the parameter α in the combined field solution should be taken $0 < \alpha \leq 1$, with a value of the order of 0.2 normally giving good results. To give some additional information on this choice, Table 1 gives the RMS error in current (Δ) and the backscattering ($\sigma/\pi a^2$) for a conducting sphere of radius a. The three values ka = 1.50, 2.75, and 4.00 were chosen for ka, and the parameter α was varied from 0.2 to 1.0. Also shown are results for the H-field solution ($\alpha = 0$) and for the E-field solution ($\alpha = \infty$). The exact values of $\sigma/\pi a^2$ are shown in the last row, computed from the spherical mode solution [4]. As is evident from the table, the value $\alpha = 0.2$ always gave good results, but the error was small for any value of α from 0.2 to 1.0.

Table 1. RMS Error in current (Δ) and Backscattering ($\sigma/\pi a^2$) for various solutions for a conducting sphere of radius a. H-field solution corresponds to $\alpha = 0$. E-field solution corresponds to $\alpha = \infty$.

	RMS error in (Δ)			Backscattering ($\sigma/\pi a^2$)		
ka =	1.50	2.75	4.00	1.50	2.75	4.00
H-field	0.0132	0.1342	0.0243	1.076	0.8702	0.7274
α= 0.2	0.0118	0.0164	0.0274	1.075	0.8645	0.7842
α= 0.4	0.0117	0.0187	0.0362	1.070	0.8648	0.7821
α= 0.6	0.0127	0.0211	0.0447	1.065	0.8647	0.7912
α= 0.8	0.0141	0.0233	0.0516	1.060	0.8645	0.8040
α= 1.0	0.0154	0.0254	0.0571	1.057	0.8637	0.8159
E-field	0.0514	0.5991	0.1894	1.043	0.8246	0.8586
Exact	0	0	0	1.076	0.8552	0.7853

The problem of scattering by a conducting right circular cylinder of cross sectional radius a and height 2a was considered to illustrate further

the effect of internal resonance on the various solutions. The excitation is a plane wave axially incident on the flat end of the cylinder. The first internal resonance varying as cos ϕ occurs at ka = 2.42 approximately. Figure 1 shows the normalized backscattering cross section (σ/λ^2) in the vicinity of the first internal resonance. The range of ka is from 2.37 to 2.47 in increments of 0.005. Note that the internal resonance had no observable effect on the combined field solution with α = 0.3 (triangles), a small effect on the E-field solution (circles), and a larger effect on the H-field solution (squares).

To estimate the RMS error in the current, the combined field solution was considered to be the exact solution. The RMS error is then defined as

$$\Delta = \left[\frac{\iint |\underset{\sim}{J} - \underset{\sim}{J}^c|^2 \, ds}{\iint ds} \right]^{1/2} \tag{17}$$

where $\underset{\sim}{J}^c$ is combined field solution and the integration is over the surface of the scatterer. Figure 2 shows Δ for the right circular cylinder vs. ka in the vicinity of the first internal resonance. Note that the greatest RMS error occurs in the E-field solution (circles), with a somewhat smaller error in the H-field solution (squares). However, since the H-field eigencurrents radiate external to the cylinder, and the E-field eigencurrents do not (at least theoretically), the H-field backscattering cross section suffers a greater error at resonance than does the E-field backscattering cross section (Fig. 1).

In the E-field solution a resonance phenomena sometimes occurs that is unrelated to any internal resonance. For example, in the case of a sphere there was a small peak in Δ at about ka = 1.3 (Figure 5, reference [1]). In that case the resonance effect was too small to show up in the backscattering cross section (Figure 6, reference [1]). This E-field "false resonance" effect was also found to occur in the case of scattering by a flat-back cone. The cone geometry is shown in Fig. 3. The cone geometry is shown in Fig. 3. The excitation is a plane wave axially

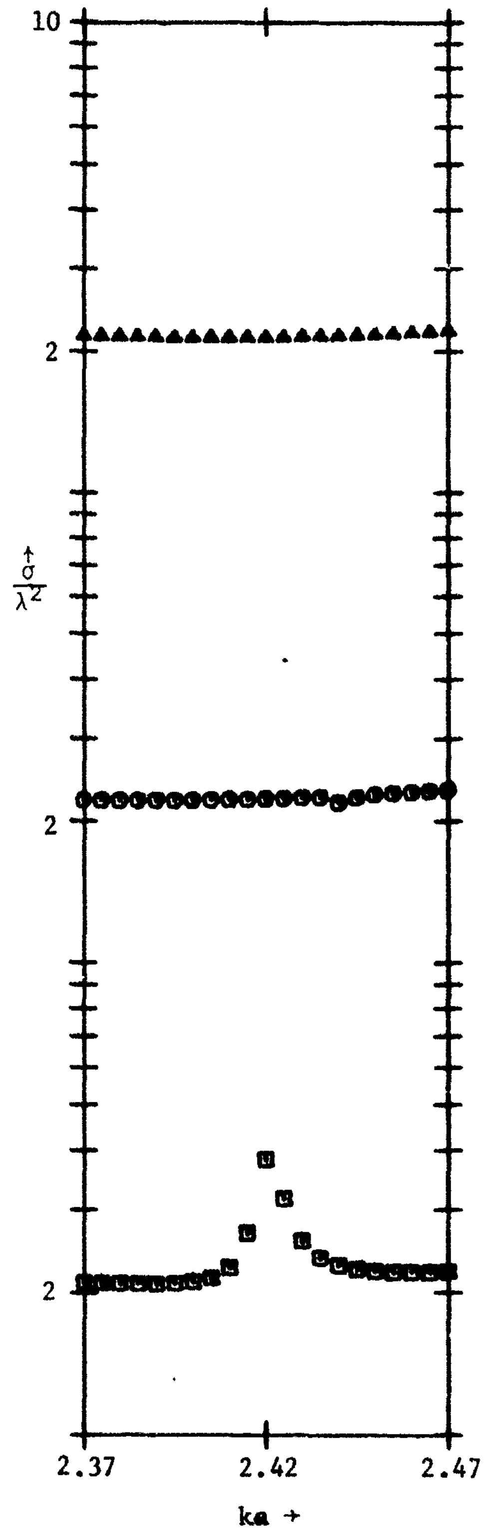

Fig. 1. Normalized radar cross section (σ/λ^2) vs ka in the vicinity of the first cos ϕ internal resonance (ka = 2.42) for a closed conducting circular cylinder of radius a and height 2a. The H-field solution is shown by squares, the E-field solution by circles, and the combined field solution with α = 0.3 by triangles. The excitation is an axially incident plane wave.

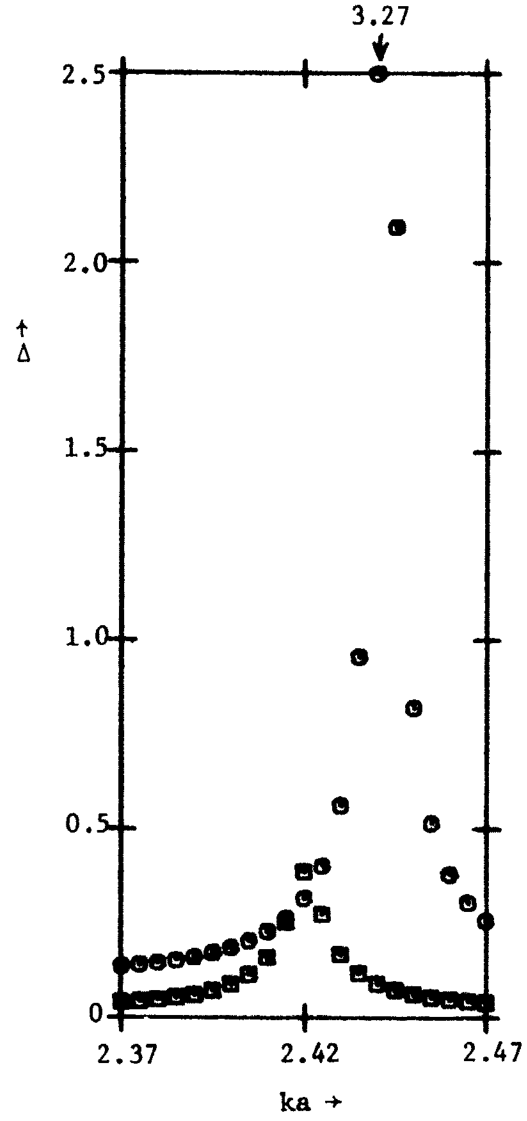

Fig. 2. RMS error (Δ) in current vs. ka in the vicinity of the first cos ϕ internal resonance (ka = 2.42) for a closed conducting circular cylinder of radius a and height 2a. The combined field solution with $\alpha = 0.3$ was assumed to be the exact solution. The H-field solution is shown by squares and the E-field solution by circles. The excitation is an axially incident plane wave.

incident on the tip of the cone. Figure 4 shows the normalized cross section (σ/λ^2) vs. frequency in the vicinity of an E-field false resonance, which occurred at 0.9 f_o, where f_o is the frequency at which $a = 3\lambda/16$. Note that the E-field solution (circles) suffers a considerable perturbation at this false resonance, while the H-field solution (squares) and the combined-field solution (triangles) are unaffected by it. Again α is taken to be 0.3 in the combined field solution. It should also be noted that the H-field solution differs from the combined field solution by an appreciable amount in this range of frequencies.

Figure 5 shows the RMS error in current for the E-field and H-field solutions in the vicinity of the false resonance for the flat-back cone. Again the combined field solution with $\alpha = 0.3$ is taken to be the exact solution, with Δ computed according to (17). Note that there is no resonance effect apparent in the H-field solution, even though there is a large effect in the E-field solution. It is felt that this false resonance is a consequence of energy storage due to the current being approximated by triangular expansion functions. If this is a correct conjecture, the resonance would probably not be present at the same frequency if the expansion functions were changed, but might occur at a different frequency.

In summary, we repeat that for the combined field solution α should be chosen in the range $0.2 \leq \alpha \leq 1$. At internal resonances, the RMS error in current is greater for the E-field solution than for the H-field solution. However, the scattering cross section error is usually smaller for the E-field solution than for the H-field solution. No effects due to internal resonances have ever been observed in the combined-field solution. Finally, the E-field solution sometimes exhibits a false resonance effect, giving incorrect solutions at some frequency which does not correspond to an internal resonance.

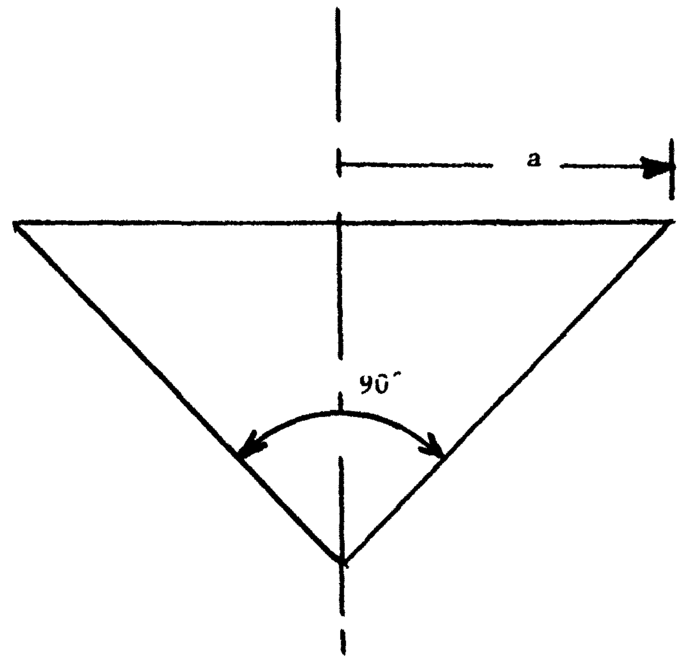

Fig. 3. A conducting flat-back cone of base radius a and cone angle 90°.

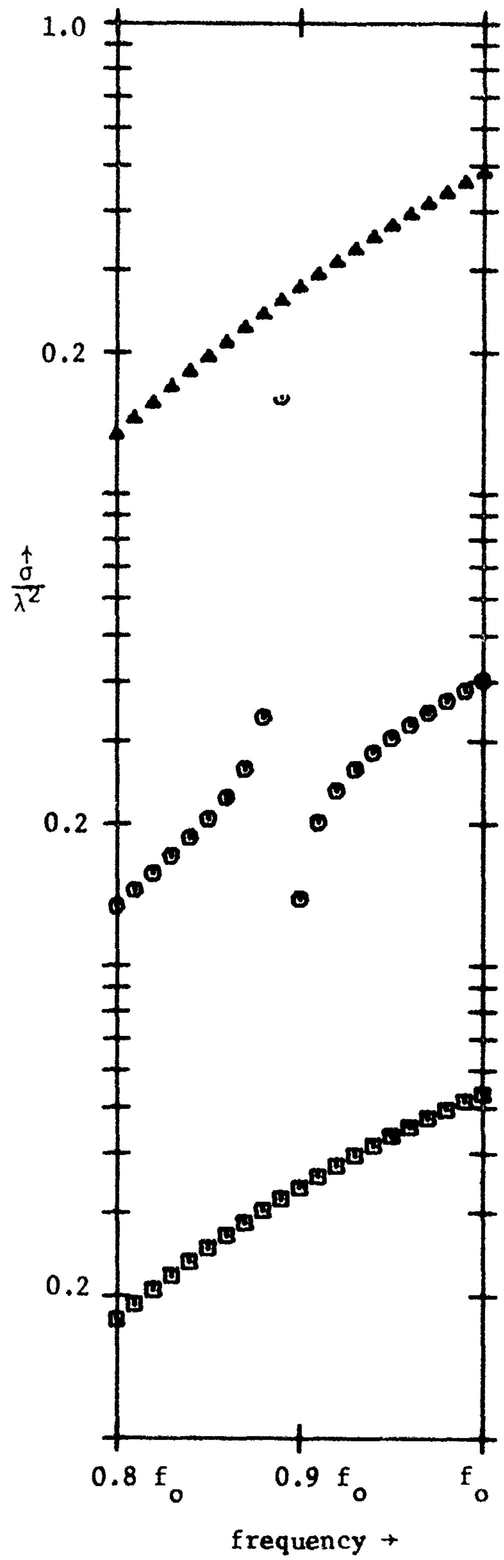

Fig. 4. Normalized radar cross section (σ/λ^2) vs. frequency in the vicinity of an E-field solution false resonance ($a = 3\lambda/16$ at f_o) for a flat-back cone of radius a and 90° cone angle. The H-field solution is shown by squares, the E-field solution by circles, and the combined field solution with $\alpha = 0.3$ by triangles. The excitation is a plane wave axially incident on the cone tip.

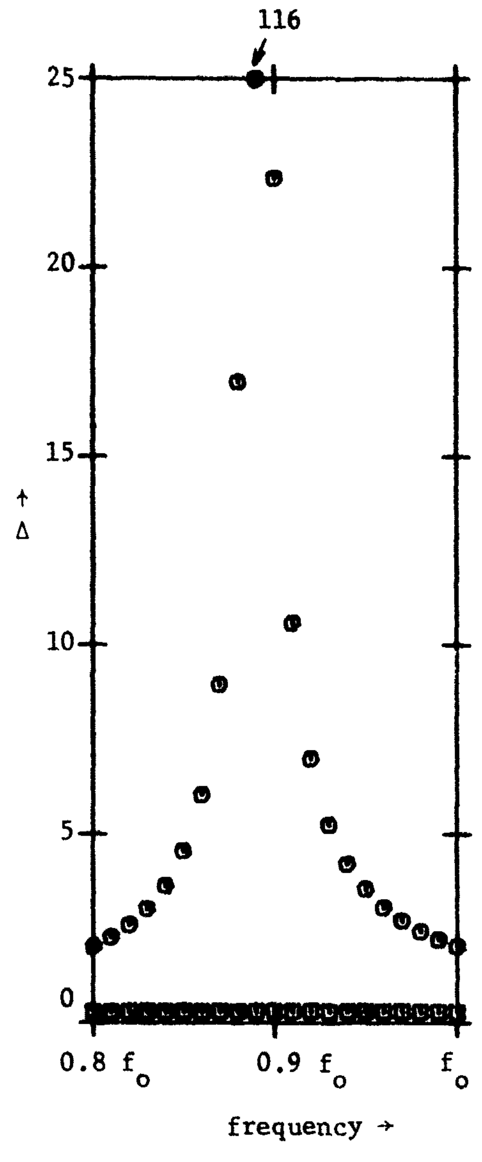

Fig. 5. RMS error (Δ) in current vs. frequency in the vicinity of an E-field solution false resonance ($a = 3\lambda/16$ at f_o) for a flat-back cone of radius a and 90° cone angle. The combined field solution with $\alpha = 0.3$ was assumed to be the exact solution. The H-field solution is shown by squares and the E-field solution by circles. The excitation is a plane wave axially incident on the cone tip.

REFERENCES

[1] J. R. Mautz and R. F. Harrington, "H-Field, E-Field and Combined Field Solutions for Bodies of Revolution," Interim Technical Report RADC-TR-77-109, Rome Air Development Center, Griffiss Air Force Base, New York, March 1977.

[2] M. Abramowitz and I. A. Stegun, "Handbook of Mathematical Functions," U. S. Government Printing Office, Washington, D.C. (Natl. Bur. Std. U. S. Applied Math. Ser. 55), 1964.

[3] J. R. Mautz and R. F. Harrington, "Transmission from a Rectangular Waveguide into Half Space Through a Rectangular Aperture," Interim Technical Report No. 12, Rome Air Development Center, Griffiss Air Force Base, New York, August 1976.

[4] S. B. Adler and R. S. Johnson, "New Backscattering Computation and Tables for Dielectric and Metal Spheres," Applied Optics, vol. 1, No. 5, pp. 655-660, September 1962.

METRIC SYSTEM

BASE UNITS:

Quantity	Unit	SI Symbol	Formula
length	metre	m	...
mass	kilogram	kg	...
time	second	s	...
electric current	ampere	A	...
thermodynamic temperature	kelvin	K	...
amount of substance	mole	mol	...
luminous intensity	candela	cd	...

SUPPLEMENTARY UNITS:

Quantity	Unit	SI Symbol	Formula
plane angle	radian	rad	...
solid angle	steradian	sr	...

DERIVED UNITS

Quantity	Unit	SI Symbol	Formula
Acceleration	metre per second squared	...	m/s
activity (of a radioactive source)	disintegration per second	...	(disintegration)/s
angular acceleration	radian per second squared	...	rad/s
angular velocity	radian per second	...	rad/s
area	square metre		m
density	kilogram per cubic metre	.	kg/m
electric capacitance	farad	F	A·s/V
electrical conductance	siemens	S	A/V
electric field strength	volt per metre	...	V/m
electric inductance	henry	H	V·s/A
electric potential difference	volt	V	W/A
electric resistance	ohm		V/A
electromotive force	volt	V	W/A
energy	joule	J	N·m
entropy	joule per kelvin	...	J/K
force	newton	N	kg·m/s
frequency	hertz	Hz	(cycle)/s
illuminance	lux	lx	lm/m
luminance	candela per square metre	..	cd/m
luminous flux	lumen	lm	cd·sr
magnetic field strength	ampere per metre	...	A/m
magnetic flux	weber	Wb	V·s
magnetic flux density	tesla	T	Wb/m
magnetomotive force	ampere	A	...
power	watt	W	J/s
pressure	pascal	Pa	N/m
quantity of electricity	coulomb	C	A·s
quantity of heat	joule	J	N·m
radiant intensity	watt per steradian		W/sr
specific heat	joule per kilogram-kelvin	.	J/kg·K
stress	pascal	Pa	N/m
thermal conductivity	watt per metre-kelvin	.	W/m·K
velocity	metre per second	.	m/s
viscosity, dynamic	pascal-second		Pa·s
viscosity, kinematic	square metre per second		m/s
voltage	volt	V	W/A
volume	cubic metre	.	m
wavenumber	reciprocal metre	...	(wave)/m
work	joule	J	N·m

SI PREFIXES:

Multiplication Factors	Prefix	SI Symbol
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto*	h
10 = 10^{1}	deka*	da
0.1 = 10^{-1}	deci*	d
0.01 = 10^{-2}	centi*	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p
0.000 000 000 000 001 = 10^{-15}	femto	f
0.000 000 000 000 000 001 = 10^{-18}	atto	a

* To be avoided where possible

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